

Homework 4 Solutions  
Stats 104

- 4.46 a) The wheel is not affected by its past outcomes – it has no memory. Outcomes are independent. So, on any one spin, black and red remain equally likely. (1 point)  
 b) Removing a card changes the composition of the remaining deck, so successive draws are not independent. If you hold five red cards, the deck now contains five fewer red cards, so your chance of red decreases. (1 point)

4.79

$$P[X \geq 26] = 1 - 0.00183 - \dots - 0.00193 = 0.99058 \quad (1 \text{ point})$$

$$\text{Mean} = -99750 \times 0.00183 - 99500 \times 0.00186 + \dots + 1250 \times 0.99058 = \$303.353$$

(1 point)

4.80

Insuring many people, the average expected value for many people will be close to \$303.353 per person so the insurance company will likely not lose money. (1 point)

4.81

X-mean	-100053	-99803	-99553	-99303	-99053	946
(X-Mean) <sup>2</sup>	10010673346	9960709170	9910869994	9861155818	9811566641	896141
*Prob	18319532	18526919	18731544	18834807	19835323	887699

$$\text{Variance} = 94,236,826 \quad (1 \text{ point})$$

$$\text{Standard deviation} = 9707 \quad (1 \text{ point})$$

4.82

a.

$$\mu_z = .5\mu_x + .5\mu_y = \mu_x$$

$$\sigma_z = \sqrt{.5^2 \sigma_x^2 + .5\sigma_y^2} = .5\sqrt{2\sigma_x^2} = \sigma_x / \sqrt{2} = 0.707\sigma_x \quad (2 \text{ points})$$

- b. If we consider  $X_1 + X_2$  to be X in part (a) and  $X_3 + X_4$  to be Y, we can see that the mean of Z is the same as the mean of X. The standard deviation will be 0.707 times the standard deviation of the result in (a), or about 0.49 the original standard deviation. (1 point)

4.83

His new mean is  $0.8\mu_w + 0.2\mu_y = 0.8(1.14) + 0.2(1.59) = 1.23$ . His new standard

$$\text{deviation is } \sqrt{0.8^2 \sigma_w^2 + 0.2^2 \sigma_y^2 + 2\rho_{wy}(0.8)\sigma_w(0.2)\sigma_y} = 4.584 \quad (2 \text{ points})$$

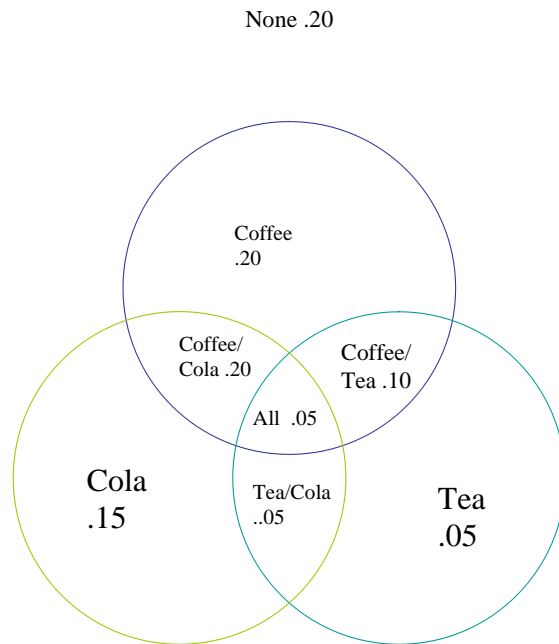
4.84

With zero correlation, the standard deviation is 3.95. The mean is unaffected by correlation. (1 point)

4.96

a) 15% drink only cola

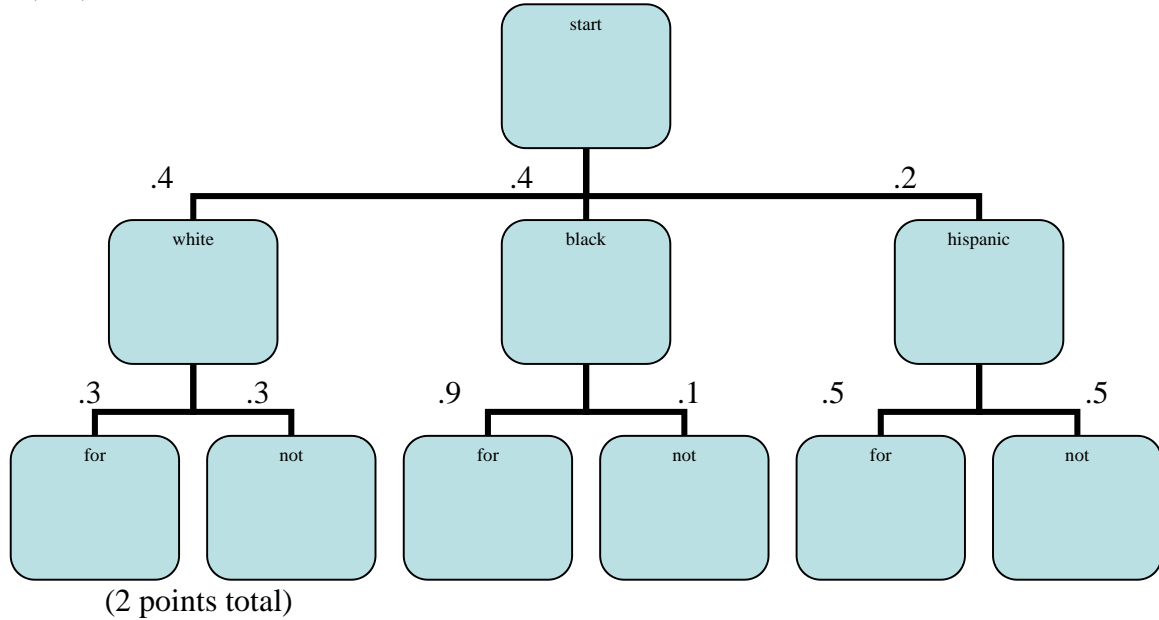
b) 20% drink none of these



(3 points total)

4.102

$$P(\text{For}) = .4 \cdot .3 + .4 \cdot .9 + .2 \cdot .5 = .58$$



4.110

Choosing surgery,  $P(A) = 0.10(0.73) + 0.85(0.76) = 0.719$ . Surgery is slightly better than medical management, where  $P(A) = 0.7$ . (1 point + 1 point for tree diagram)

Additional Problems

1.

	Farmer	Merchant	Civil Servant	Laborer	
Moslem	.21	.06	.06	.09	.42
Buddhist	.42	.08	.03	.05	.58
	.63	.14	.09	.14	

(2 points)

2.

$$P(\text{Buddhist} \mid \text{farmer}) = .42/.63 = .67$$

$$P(\text{Moslem} \mid \text{farmer}) = .21/.63 = .33$$

They add up to one since they are complements since you can only be a Buddhist or Moslem according to this table. (2 points)

3.

$$P(\text{farmer} \mid \text{Buddhist}) = .42/.58 = .72$$

$$P(\text{farmer} \mid \text{Moslem}) = .21/.42 = .5$$

They do not add up to one and they should not because they are not complements. (2 points)

4.

$$P(\text{Moslem}) = .42$$

$$P(\text{Moslem} | \text{Farmer}) = .21/.63 = .33$$

Thus they are not independent. (2 points)

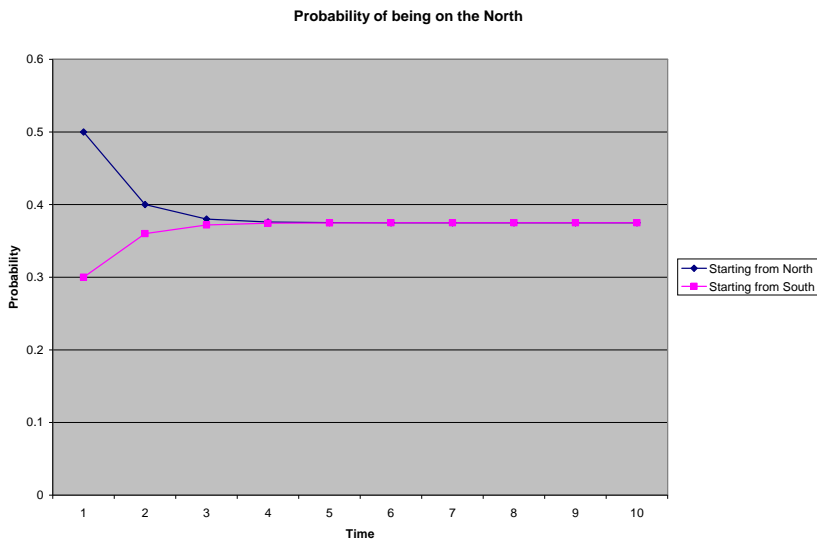
#### Markov Chain

1.  $P(\text{On north side after one minute}) = .5$

$$P(\text{On north side after two minutes}) = .5^2 + .5*.3 = .4 \quad (1 \text{ point})$$

2.

Minute	Prob on North starting from North	Prob on North starting from South
1	0.5	0.3
2	0.4	0.36
3	0.38	0.372
4	0.376	0.3744
5	0.3752	0.37488
6	0.37504	0.374976
7	0.375008	0.374995
8	0.375002	0.374999
9	0.375	0.375
10	0.375	0.375



(2 points)

3. Yes it is approaching a limit and .375 seems to be it.

(1 point)

4.  $P_E = P_E * (1 - P_S) + (1 - P_E) * P_N$   
 $P_E = P_E(1 - P_S - P_N) + P_N$   
 $P_E(P_S + P_N) = P_N$   
 $P_E = P_N / (P_N + P_S)$  (1 point)