Statistics 110 – Assignment 6

Due: Thursday, August 10, 2006

1. Rice 4.94

2. Rice 4.98

3. Rice 5.5 (Hint: use the result $(1+\frac{a}{n})^n \to e^a$)

4. Rice 5.12

5. Rice 5.14

6. Rice 5.23

7. Rice 5.24

8. Let $X_1, X_2, \ldots$ denote an iid random sample from a distribution with cumulative distribution function $F(x)$. The sample cumulative distribution function, denoted by $F_n(x)$ is defined by

$$F_n(x) = \frac{1}{n} \times [\# \text{ of } X_1, \ldots, X_n \leq x]$$

Show that for a fixed x where x is a continuity point of $F(x)$, $F_n(x) \xrightarrow{P} F(x)$. (Hint: What is distribution of $F_n(x)$?)

9. Consider a Markov chain on states {1,2,3,4,5,6}. Suppose the transition probability matrix is

(a)

$$\begin{bmatrix} 1/3 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 3/4 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 4/5 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 1/8 & 7/8 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/8 & 3/8 & 0 \\ 1/3 & 0 & 1/6 & 1/4 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For each case, find all communicating classes. Also which classes are recurrent and which are transient?

10. On any given day, Buffy is either cheerful (C), so-so (S), or gloomy (G). If she is cheerful today, then she will be C, S, or G tomorrow with respective probabilities 0.7, 0.2, 0.1. If she is so-so today, then she will be C, S, or G tomorrow with respective probabilities 0.4, 0.3, 0.3. If she is gloomy today, then she will be C, S, or G tomorrow with respective probabilities 0.2, 0.4, 0.4. What proportion of time is Buffy cheerful? What is the long-run average number of iterations between gloomy days?

11. Each of 2 switches is either on or off during a day. On day n, each switch will independently on with probability

$$[1 + \text{ number of on switches during day } n-1]/4$$

For instance, if both switches are on during day $n-1$, then each will be independently be on during day n with probability 3/4. What fraction of days are both switches on?

Suggested additional problems from Rice (don't hand in)

5.13, 5.17 (also use Chebyshev to put a lower bound on n such that $P[|\bar{X} - \mu| < 1] \geq 0.95$), 5.26